AP Calculus BC

WS 86 - Convergence of Series 2

1) $\sum_{n=1}^{\infty}\left(\frac{8}{9}\right)^n$

$S = \frac{8/9}{1-8/9} = 8$

2) I) $\sum \frac{\cos n}{2^n} = S$

AST

* $\lim_{n\to\infty} \frac{1}{2^n} = 0$
* $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

$\sum \frac{1}{2^n}$ converges by GST

S converges absolutely

II) $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{5}+1)^n} = S$

$r = \frac{1}{\sqrt{5}+1} < 1$

S converges by GST

III) $\sum_{n=1}^{\infty} \frac{2+\sin n}{n} = S$

DCT

$\sum \frac{1}{n}$ diverges b p-series

$\frac{1}{n} \le \frac{2+\sin n}{n}$

S diverges

3) $\sum \frac{n^n}{n!} = S$

Ratio Test

$\lim_{n\to\infty}\left[\frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n}\right]$

$\lim_{n\to\infty}\left[\frac{(n+1)^{n+1}}{n+1}\cdot\frac{1}{n^n}\right]$

$\lim_{n\to\infty}\left[\frac{(n+1)^{n}}{1}\cdot\frac{1}{n^n}\right]$

$\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]$

$\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n = e > 1$

S diverges

4) $\sum \frac{e^{1/n}}{n^2} = S$

$\int_1^{\infty} \frac{e^{1/x}}{x^2} = \int_1^{\infty} e^{x^{-1}} \cdot x^{-2}\,dx$

$\lim_{b\to\infty}\int_1^b e^{x^{-1}}\cdot x^{-2}\,dx = \lim_{b\to\infty}\left[-e^{x^{-1}}\right]_1^b$

$\lim_{b\to\infty}\left[-e^{1/b}+e^1\right] = -1+e$

S converges

5) a) $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = S$

$r = \frac{1}{2} < 1$

S converges by GST

b) $\sum_{n=1}^{\infty} \frac{1}{n!} = S$

$\lim_{n\to\infty}\left[\frac{1}{(n+1)!}\cdot n!\right]$

$\lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$

S converges by Ratio Test

c) $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series

d) $\sum \frac{1}{n^{3/2}}$

converges by p-series

e) $\sum \frac{1}{n^2}$

Converges by p-series

6) a) $\sum_{n=1}^{\infty} \frac{1}{\ln n} = S$

LCT

$\sum \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left[\frac{1}{\ln n} \cdot n\right] = \infty$

S diverges

b) $\sum_{n=1}^{\infty} \frac{1}{n}$

diverges by p-series

c) $\sum \frac{1}{e^n}$

Converges by GST $\therefore$

d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series

e) $\sum n$ diverges by n^{th} term test

$\lim_{n\to\infty} n = \infty$

7) a) $\sum \frac{1}{2n-1}$

$\sum \frac{1}{n}$ diverges

$\lim_{n\to\infty} \left[\frac{1}{2n-1} \cdot n\right] = \frac{1}{2} > 0$

S diverges by LCT

b) $\sum \frac{n+1}{n+2} = S$

$\lim_{n\to\infty} \frac{n+1}{n+2} = 1$

S diverges by n^{th} term test

c) $\sum \frac{3^n}{n!} = S$

$\lim_{n\to\infty} \left[\frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}\right]$

$\lim_{n\to\infty} \frac{3}{n+1} = 0 < 1$

S converges by Ratio Test

d) $\sum \frac{n^2}{n^3+1} = S$

$\sum \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left|\frac{n^2}{n^3+1} \cdot n\right| = 1 > 0$

S diverges by LCT

e) $\sum \frac{n}{2^n} = S$

$\lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \frac{1}{2} < 1$

S converges by Root Test

f) $\sum \frac{\sin n}{3^n} = S$

$\sum \frac{1}{3^n}$ converges by GST

$\frac{1}{3^n} \geq \frac{\sin n}{3^n}$

S converges by GST

g) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^2+3}} = S$

$\sum \frac{1}{n}$ diverges

$\lim_{n\to\infty} \left|\frac{1}{\sqrt{2n^2+3}} \cdot n\right| = \frac{1}{\sqrt{2}} > 0$

S diverges by LCT

h) $\sum_{n=2}^{\infty} \frac{2n}{n^2-1} = S$

LCT

$\sum \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left|\frac{2n}{n^2-1} \cdot n\right| = 2 > 0$

S diverges by LCT

8) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2}{n^3+1} = S$

✓ $\lim_{n\to 1} \frac{n^2}{n^3+1} = 0$

✓ $\frac{1}{2}, \frac{4}{9}, \frac{9}{28}$

$\sum \frac{n^2}{n^3+1}$ diverges

LCT

$\sum \frac{1}{n}$ div.

$\lim_{n\to\infty} \left|\frac{n^2}{n^3+1} \cdot n\right| = 1 > 0$

S converges conditionally